Introduction to AdaBoost

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What is AdaBoost?

- Bagging = “Parallel”
- AdaBoost = “Sequential”

- Originally **binary classification**

- Extensions for multiclass, regression, ranking, …

Ever Used AdaBoost?

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Ever played Call of Duty?

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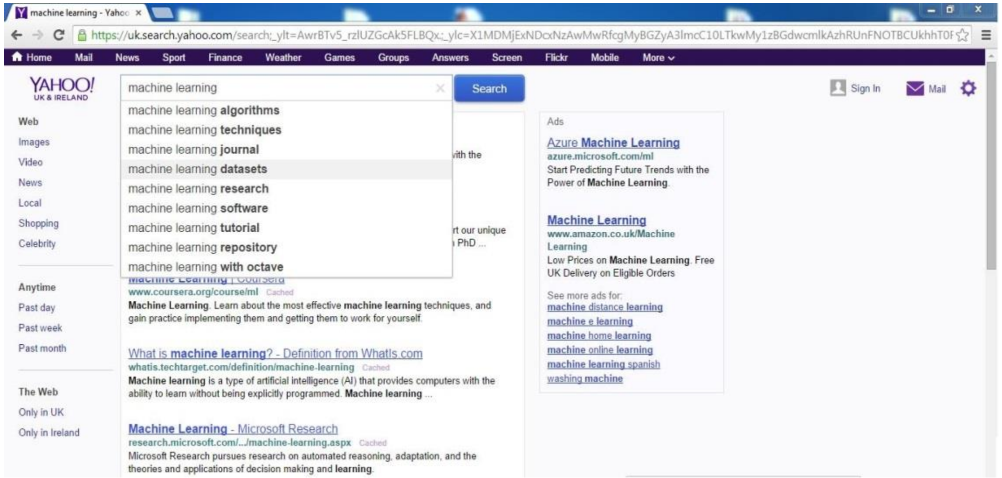

Or used the Yahoo search engine…?

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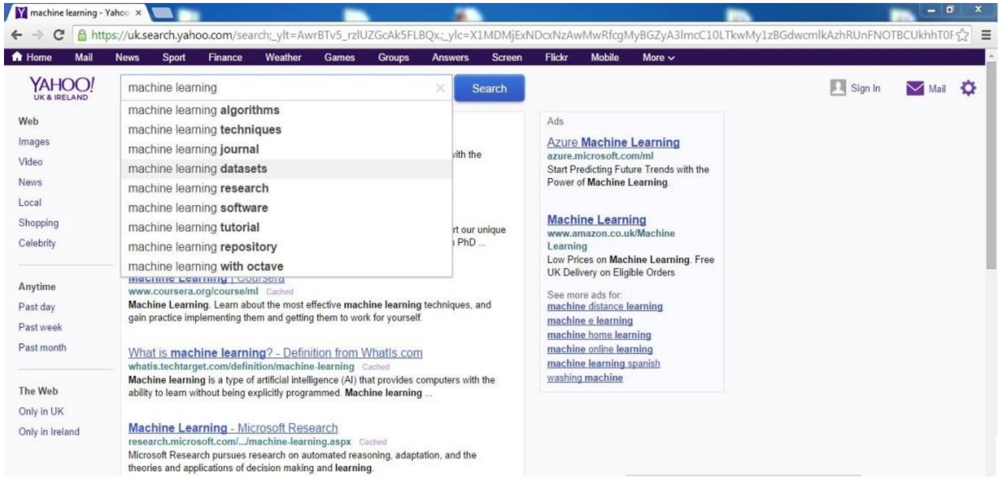

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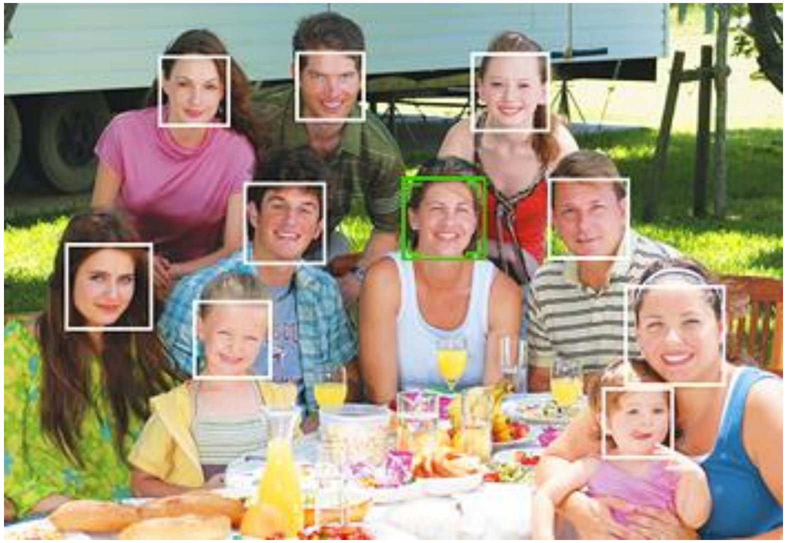

Or have a phone with a camera…?

History of the Concept (1)

- Can we turn a weak learner into a strong learner? (Kearns, 1988)

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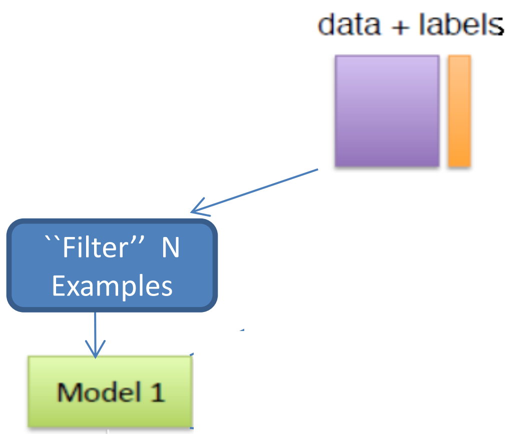

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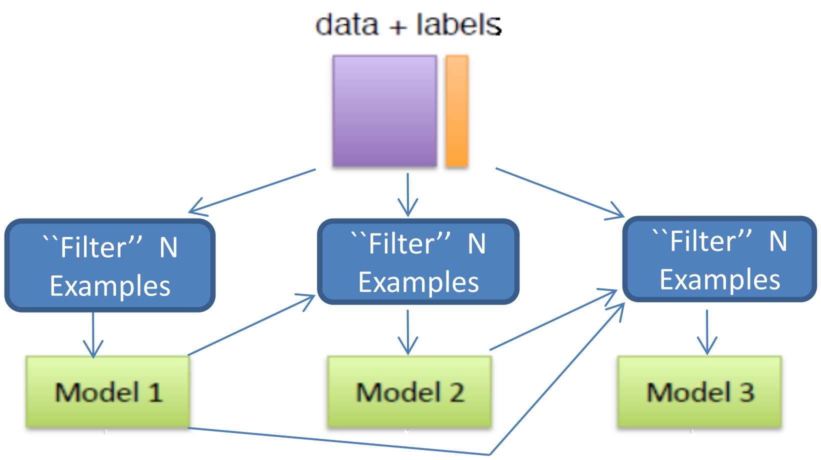

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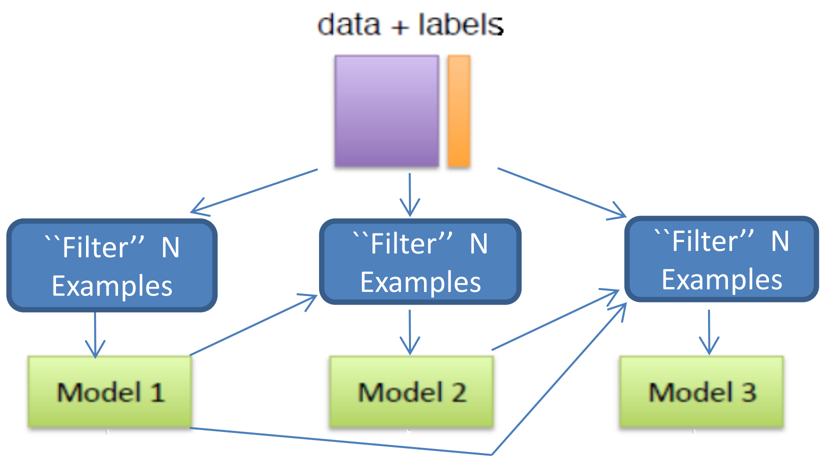

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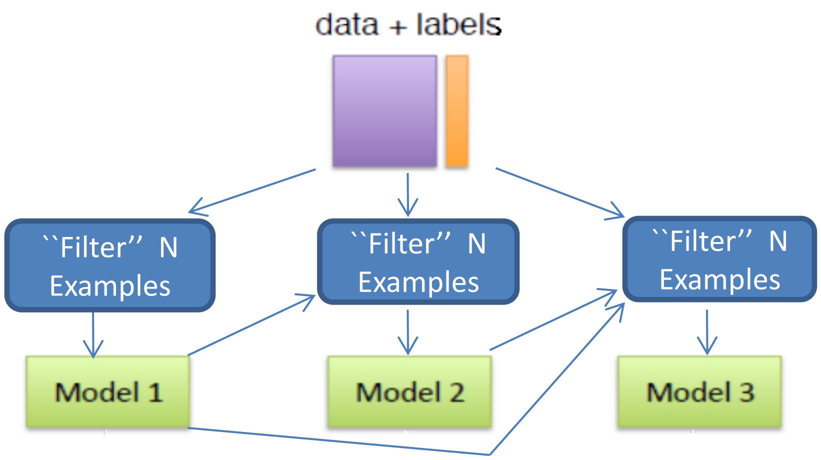

Majority Vote better than Model 1

- Adaptive boosting (**Adaboost**): M models built "adaptively" (Freund & Schapire, 1997)

2003 Gödel Prize

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kaggle™

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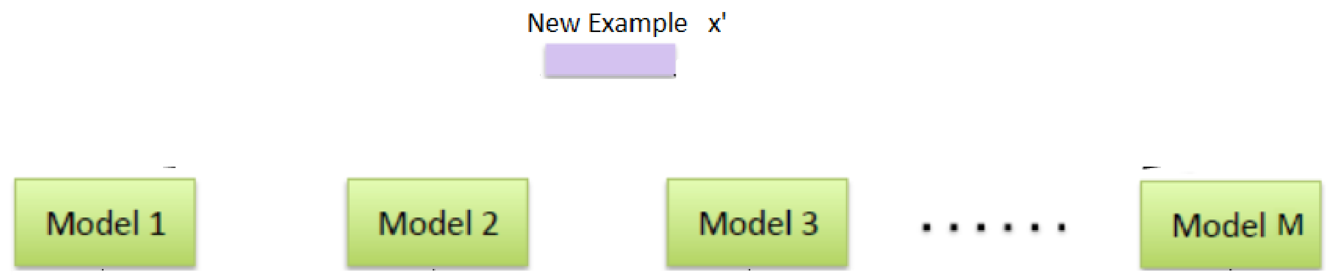

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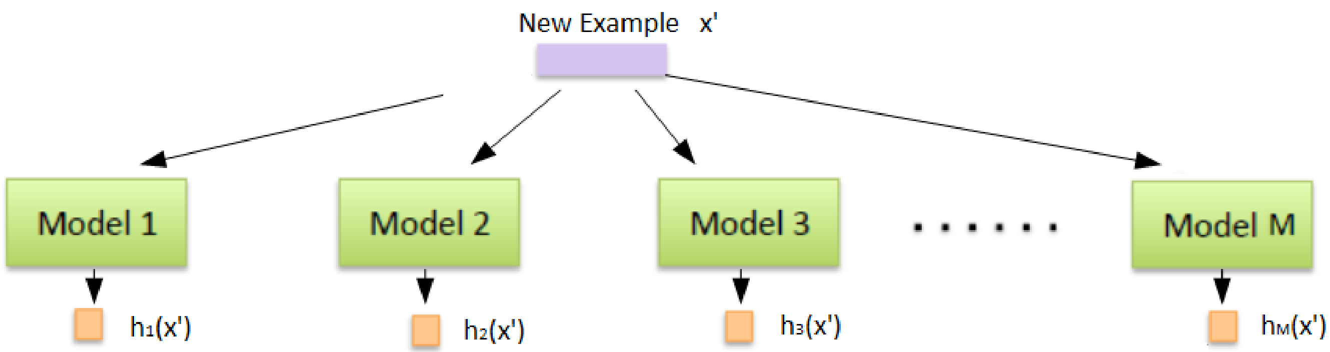

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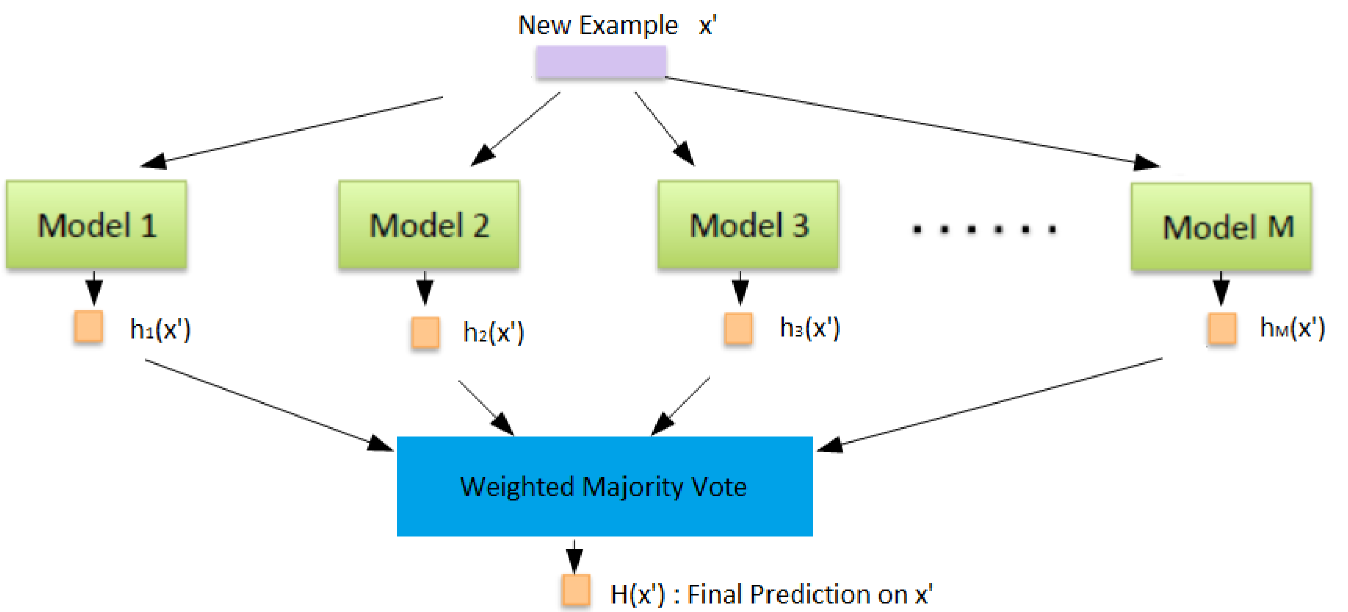

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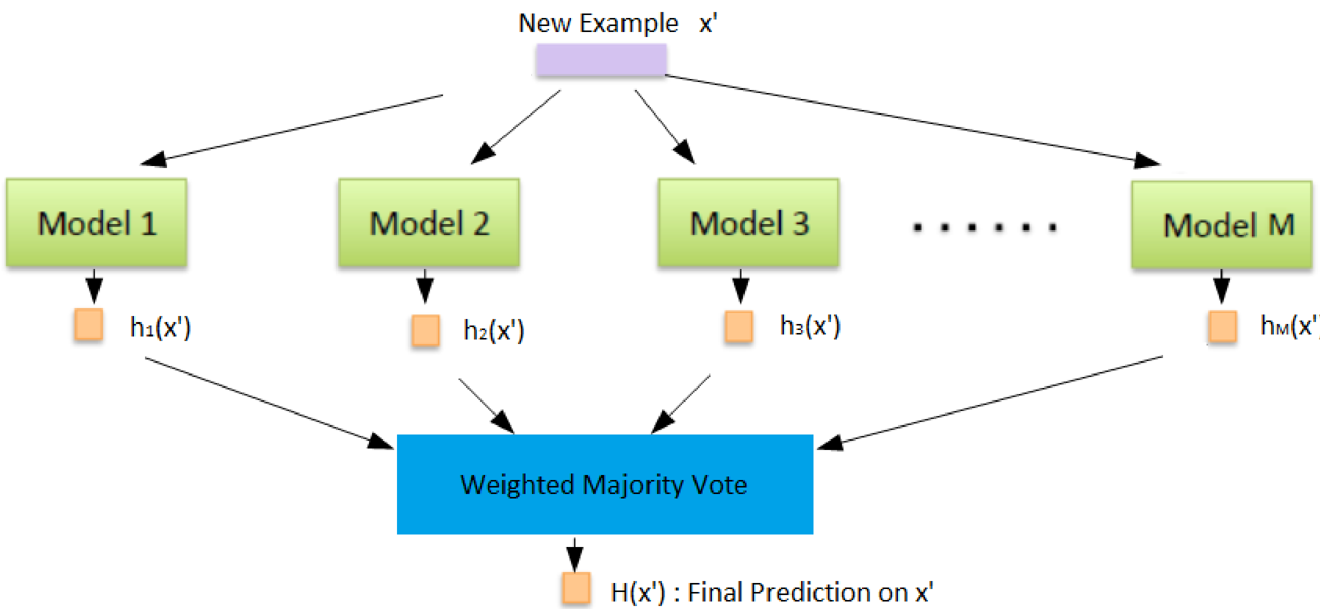

- Can apply to any supervised base learner...

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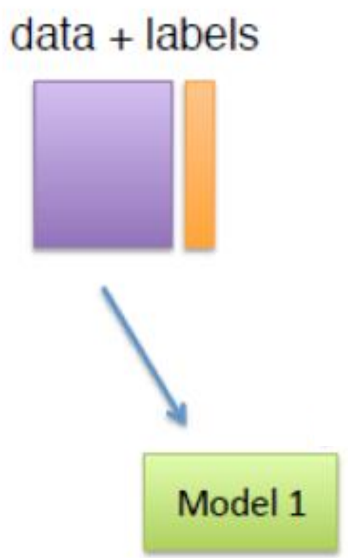

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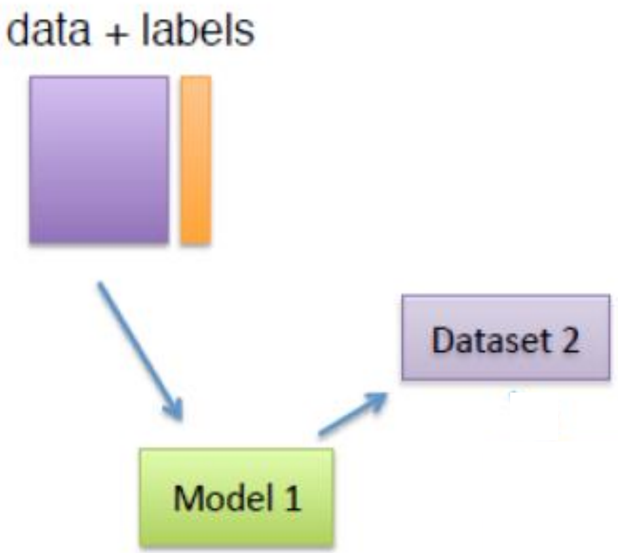

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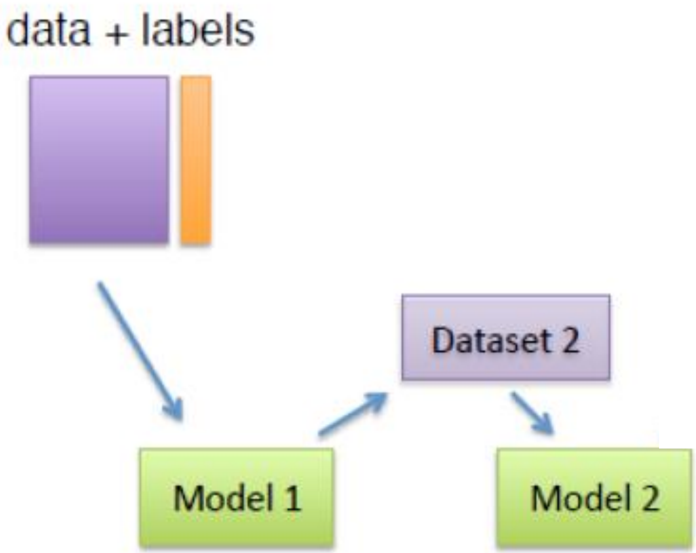

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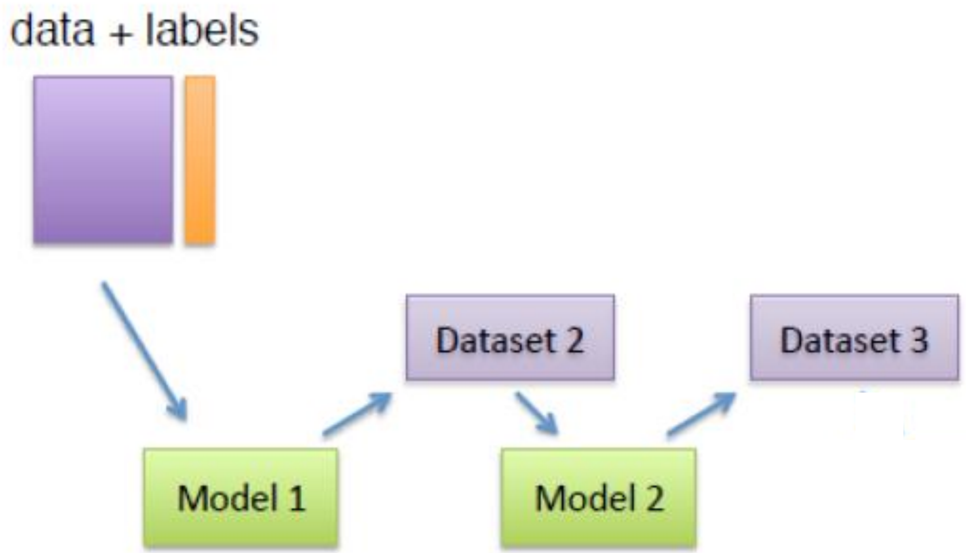

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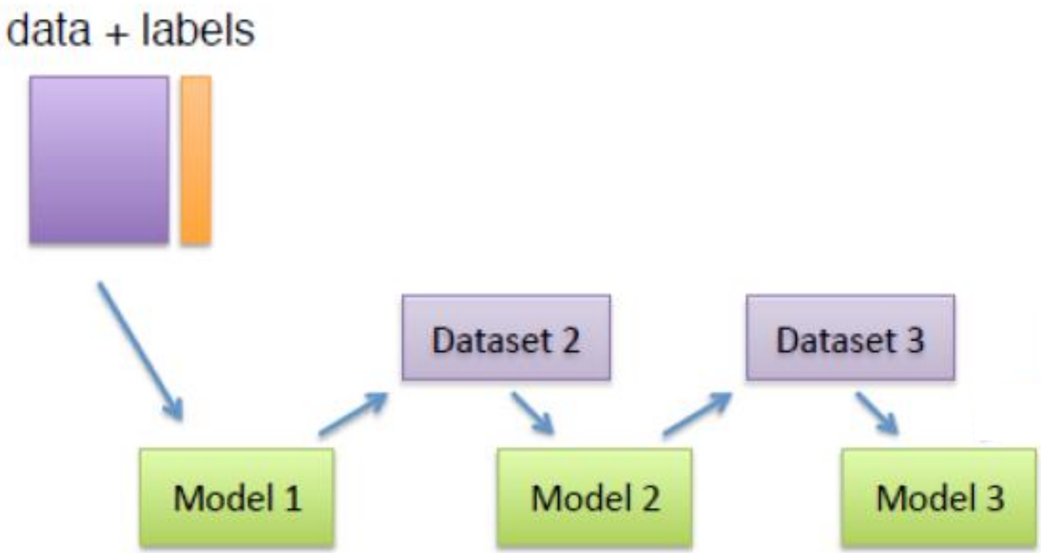

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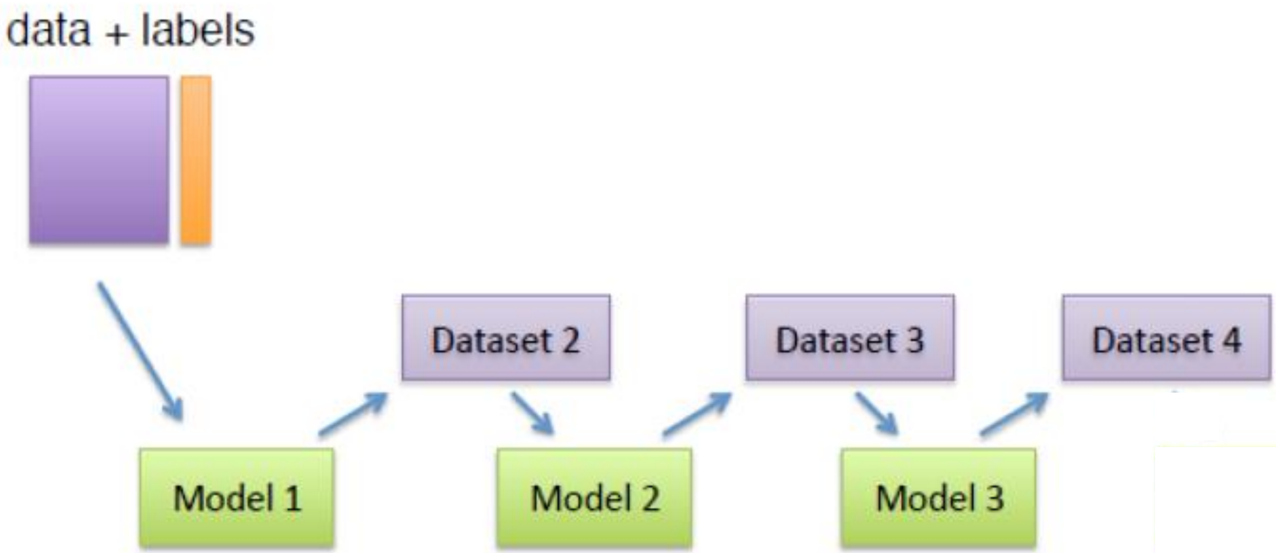

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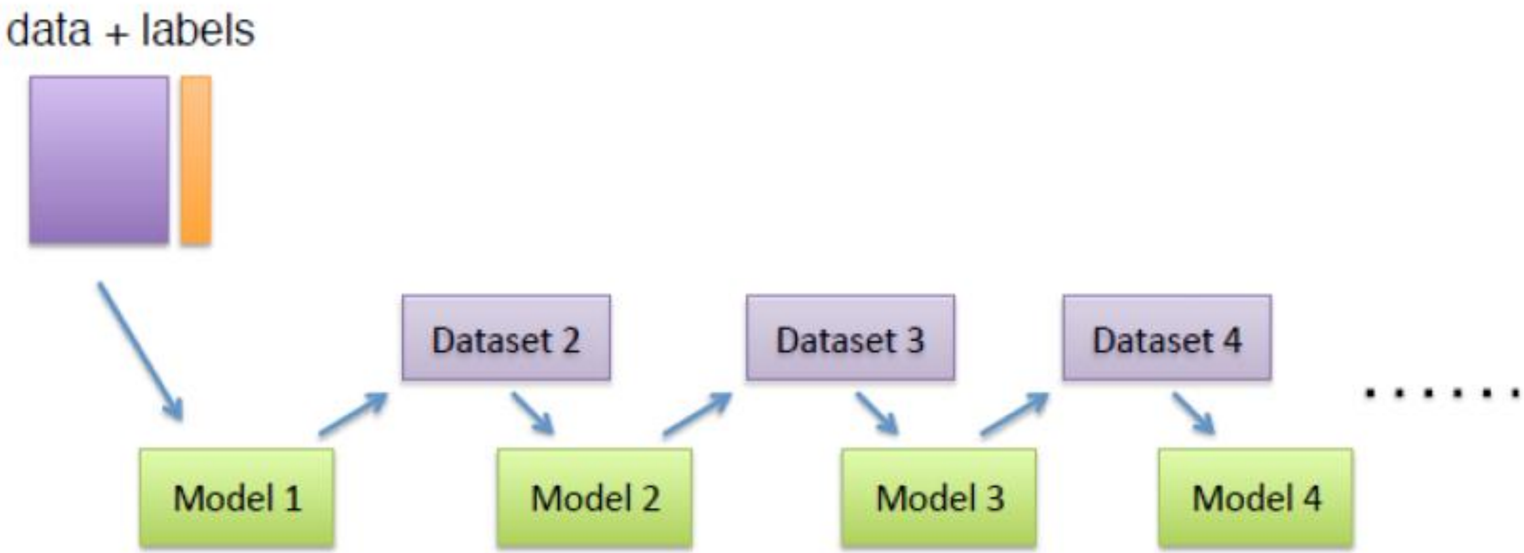

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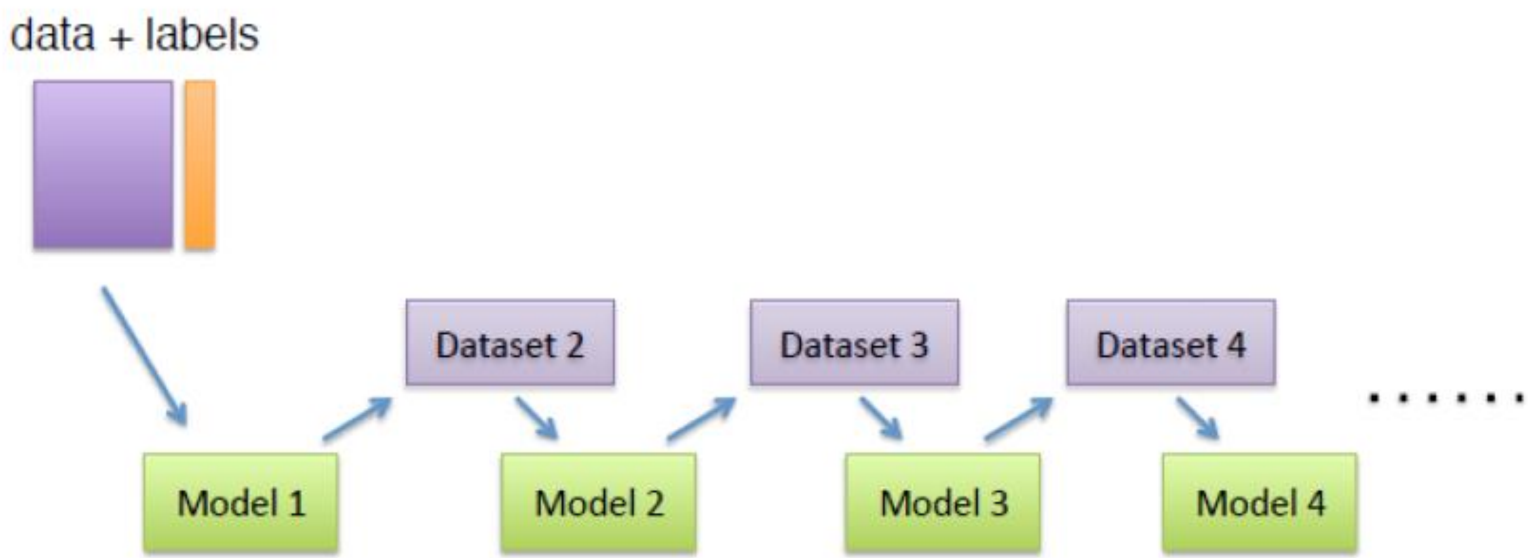

- Each model tries to **correct the mistakes of the previous one**

AdaBoost: Algorithm Outline

Algorithm 1: AdaBoost Sketch

Input: Training Data $S = \{(x_1, y_1), \ldots, (x_N, y_N)\}$, Number of rounds M.
Training:
Define a weight distribution over the examples $D_i^1 = \frac{1}{N}$, for $i = 1, 2, \ldots, N$.
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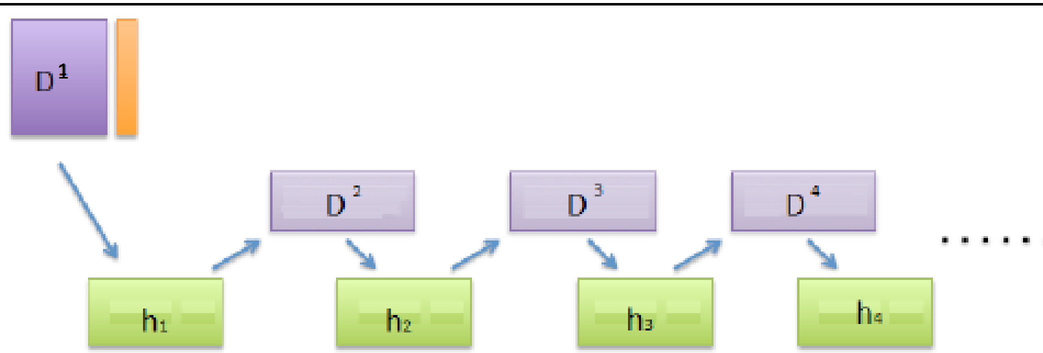

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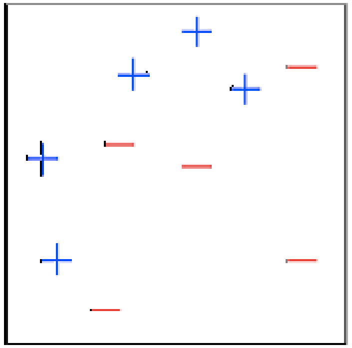

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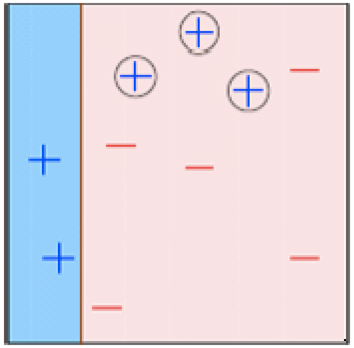

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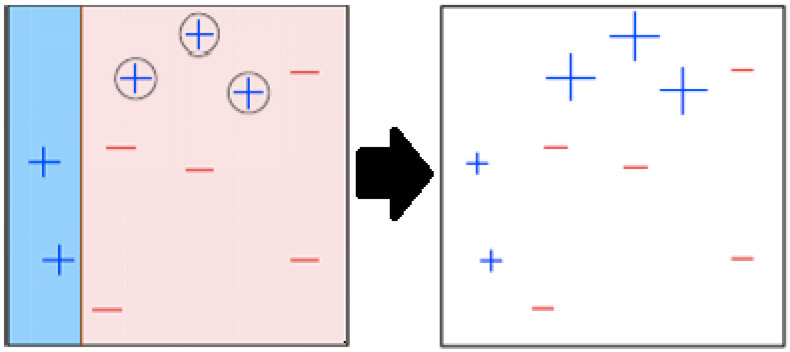

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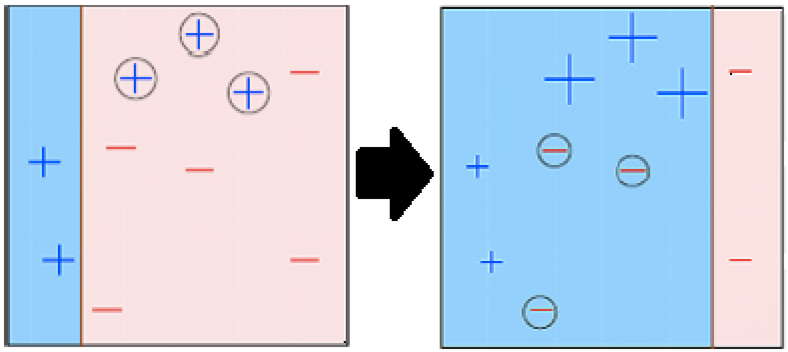

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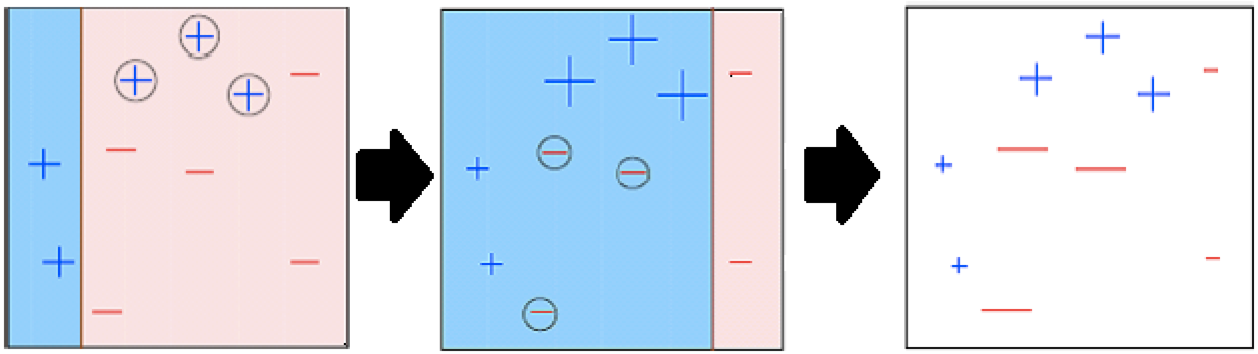

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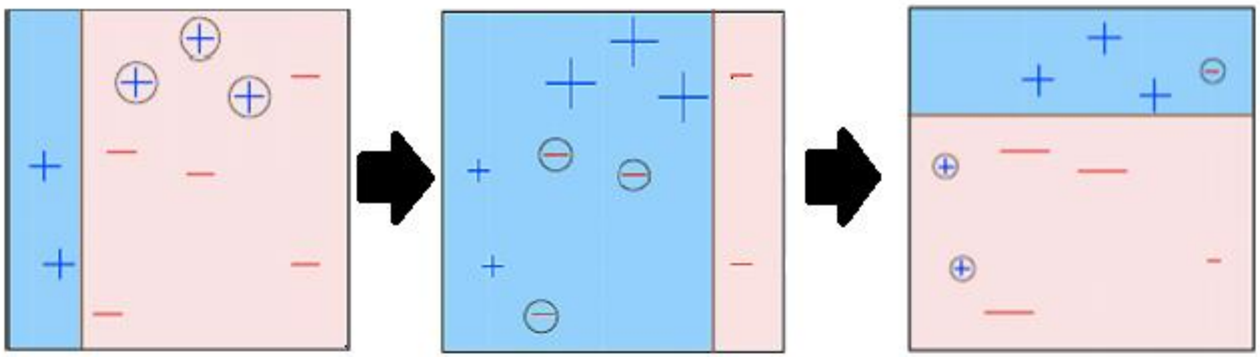

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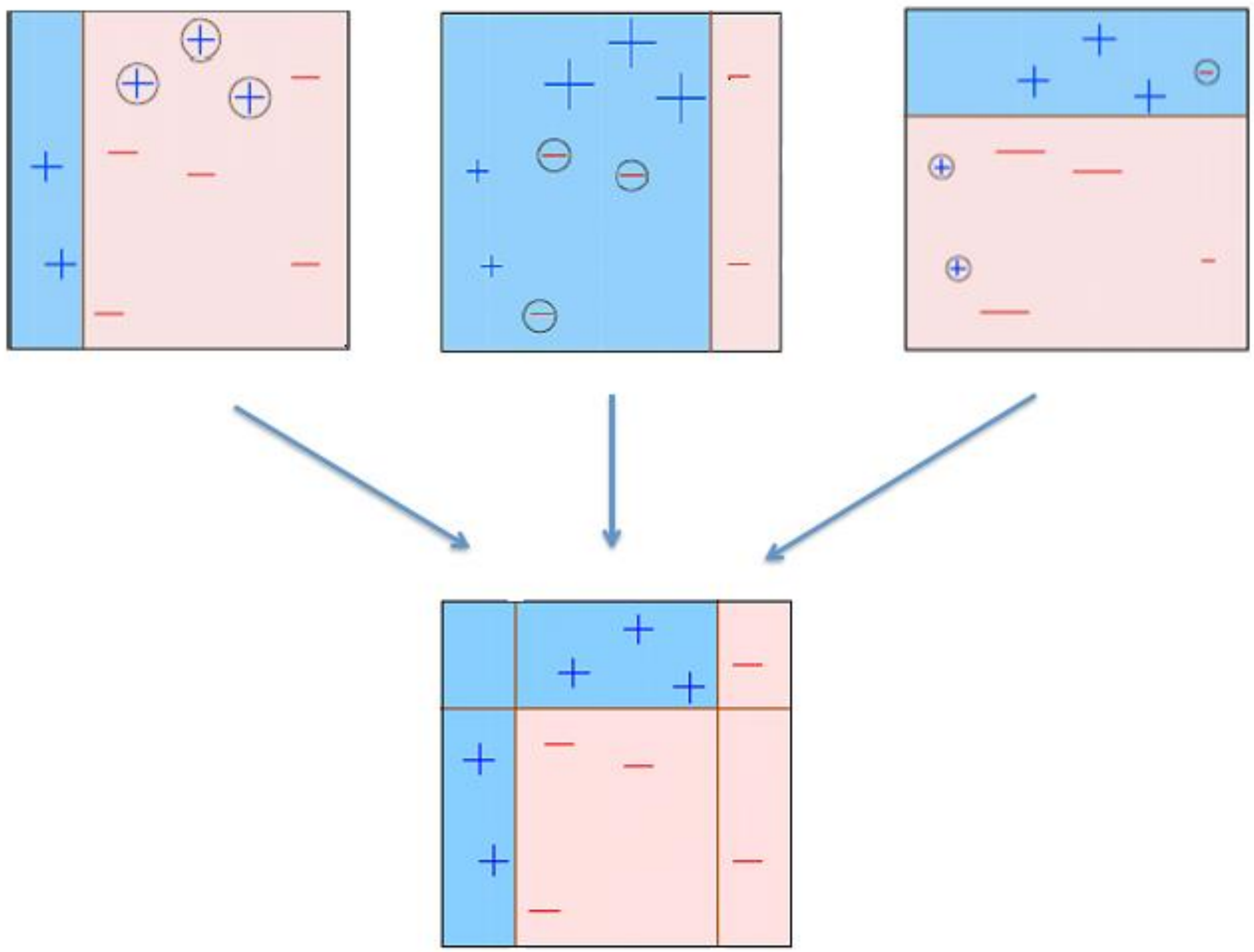

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- So, bagging & AdaBoost **fundamentally different**!

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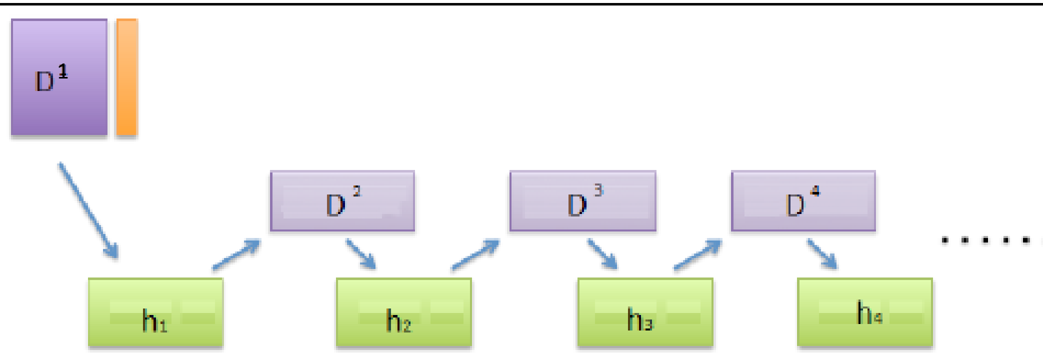

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αj: ``confidence'' of the j-th model

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ϵ_j: weighted error of the j-th model

α_j: ``confidence'' of the j-th model

Update weights for next iteration

After normalization we have: $\sum_{i=1}^{N} D_i^{j+1} = 1$

$y \in \{-1, 1\}$

AdaBoost: Prediction

- A new example **x**' arrives
- Each weak learner's vote $h_j(\mathbf{x}') \in \{-1,1\}$ is weighted by its confidence α_j to get the final prediction

$$H(\boldsymbol{x}') = sign\left[\sum_{j=1}^{M} \alpha_j h_j(\boldsymbol{x}')\right]$$

Adaboost: Training (1)

- In each training round:

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QUESTION:

What if $\epsilon_j > \frac{1}{2}$?

ANSWER: Just **flip** predictions!

Resulting learner **has** $\epsilon_j < \frac{1}{2}$

Adaboost: Training (2)

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  But $\epsilon_j < \frac{1}{2}$, so $\alpha_j > 0$

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Adaboost: Training (2)

- In each training round:

  Define $\epsilon_j = \sum_{i:h_j(\mathbf{x}_i)\neq y_i} D_i^j$.

  Obtain a hypothesis h_j that minimizes ϵ_j

  $\alpha_j = \frac{1}{2}\log\left(\frac{1-\epsilon_j}{\epsilon_j}\right)$.

  But $\epsilon_j < \frac{1}{2}$, so $\alpha_j > 0$

  $D_i^{j+1} = e^{-y_i h_j(\mathbf{x}_i)\alpha_j} D_i^j$.

Remember, $y \in \{-1, 1\}$ so, 2 cases:

$y_i = h_j(\mathbf{x}_i) \implies y_i h_j(\mathbf{x}_i) = 1 \implies D_i^{j+1} = e^{-\alpha_j} D_i^j \implies D_i^{j+1} < D_i^j$

(h_{j+1} **can afford to pay less attention** to i-th example)

OR

$y_i \neq h_j(\mathbf{x}_i) \implies y_i h_j(\mathbf{x}_i) = -1 \implies D_i^{j+1} = e^{\alpha_j} D_i^j \implies D_i^{j+1} > D_i^j$

(h_{j+1} must **try harder** than h_j to get i-th example right)

Adaboost: Training (3)

- In each training round:

  Define $\epsilon_j = \sum_{i:h_j(\mathbf{x}_i)\neq y_i} D_i^j$.

  Obtain a hypothesis h_j that minimizes ϵ_j

  $\alpha_j = \frac{1}{2}\log\left(\frac{1-\epsilon_j}{\epsilon_j}\right)$.

  $D_i^{j+1} = e^{-y_i h_j(\mathbf{x}_i)\alpha_j} D_i^j$.

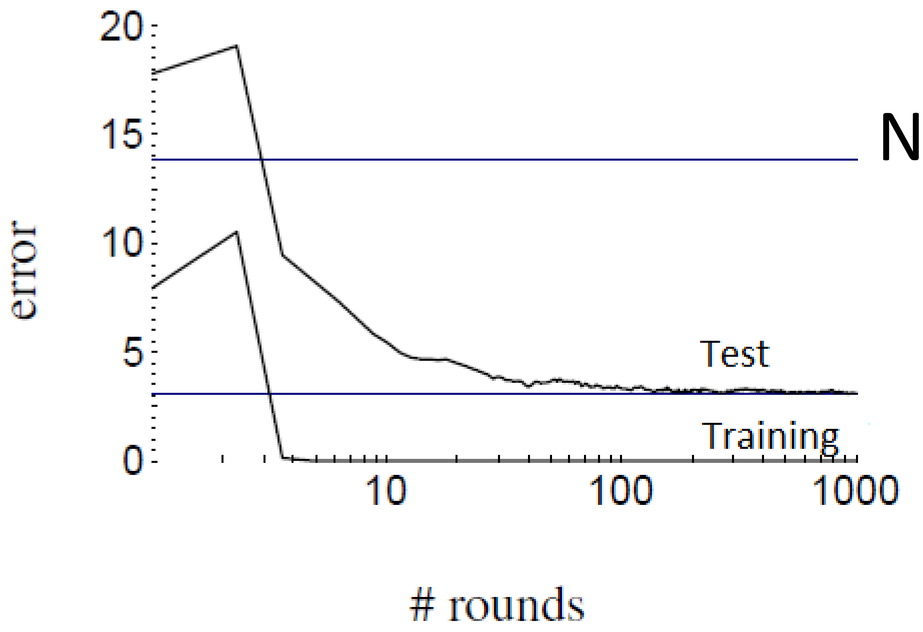
Note: weight updates are **confidence-rated**; the higher the confidence α_j , the greater the weight increase/decrease of each D_i^{j+1}

Why this α_j?

- Why the magic choice of $\alpha_j = \frac{1}{2}\log\left(\frac{1-\epsilon_j}{\epsilon_j}\right)$?
- Beyond scope of lecture
- A consequence: 50% of new weight mass D^{j+1} assigned to examples misclassified by previous learner h_j.
  - Decorrelates consecutive learners

Resistance to Overfitting

- Often we observe AdaBoost behaving like this:



Notice something strange?

Resistance to Overfitting

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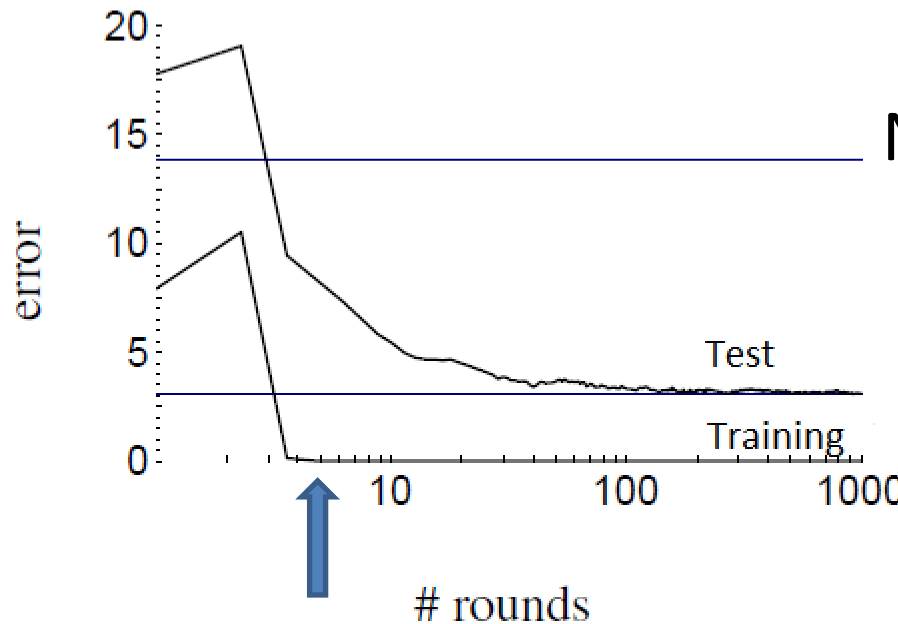

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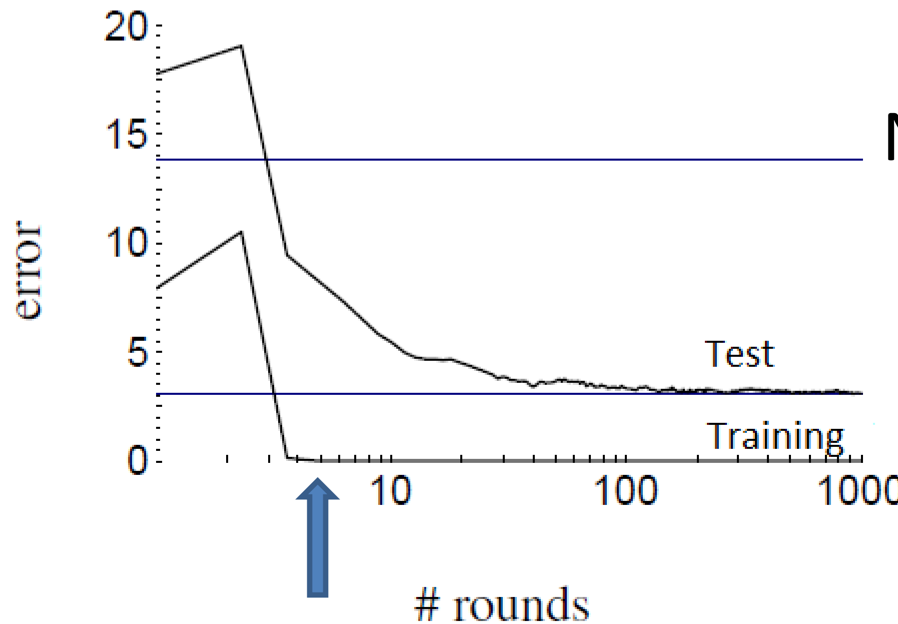

Notice something strange?

- Test error decreases even after training error reached zero!

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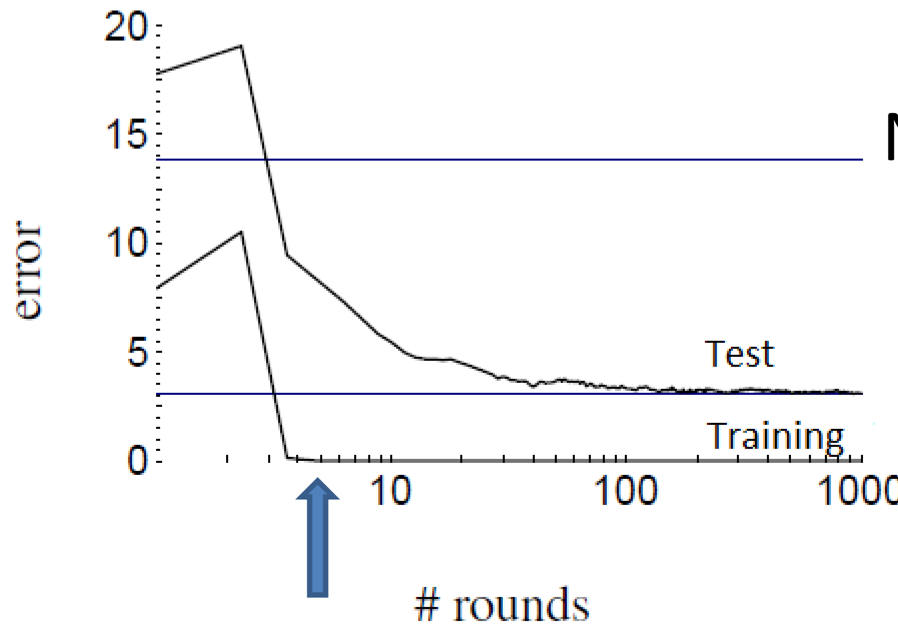
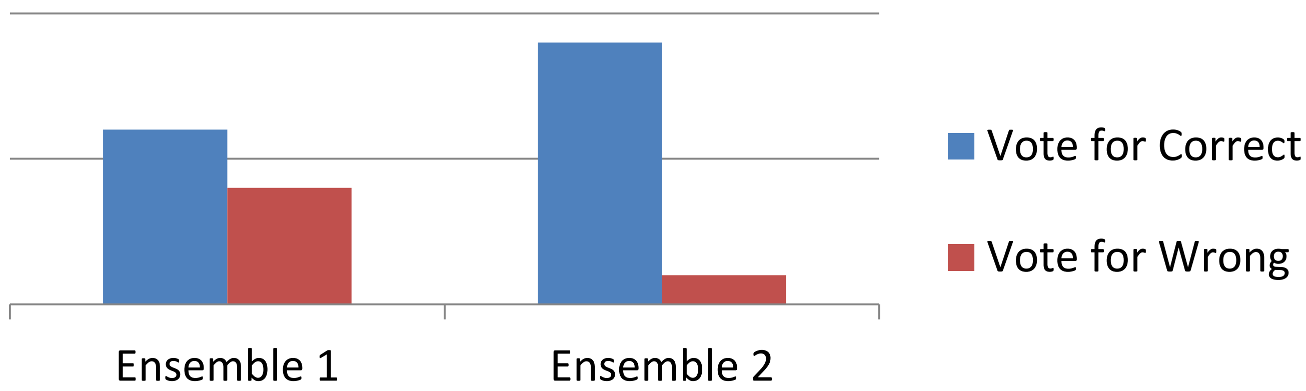


Notice something strange?

- Test error decreases even after training error reached zero!
- But… more weak learners… more complex hypothesis, how come we are not overfitting?

An Explanation: Margin Theory

- **Voting Margin = Fraction Voting Correctly – Fraction Voting Incorrectly**
- **Not SVM margin**, but related; **measures confidence** of the ensemble



- AdaBoost keeps increasing the margins even after it has managed to classify all training examples correctly (Schapire et al, 1998)

Interpretations of AdaBoost

- PAC learning (Schapire, 1990)
- Game Theory (Freund & Schapire, 1996)
- VC-Theory (Freund & Schapire, 1997)
- **Margin Theory** (Schapire et al, 1998)
- Information Theory (Kivinen & Warmuth, 1999)
- Optimizing Bregman Divergence (Collins, 2000)
- Minimization of an exponential loss (Friedman et al, 2000)
- Functional Gradient Descent (Mason et al, 2001)
- Probabilistic (Lebanon & Lafferty, 2001; Edakunni, Brown & Kovacs, 2011)
- Dynamical systems (Rudin et al, 2004)
- Implicit regularization via early stopping (Rosset et al, 2004; Zhao & Yu, 2007)

- And many more, mostly **complementary**; each explains some aspects

Strengths & Weaknesses

Strengths
Few parameters
Simple to implement
Implicit feature selection
Resistant to overfitting (when low noise)
Performs very well in practice

Strengths & Weaknesses

Strengths	Weaknesses
Few parameters	**Needs a termination condition**
Simple to implement	**Sensitive to noisy data & outliers Why?**
Implicit feature selection	**Must adjust for cost-sensitive or imbalanced class problems**
Resistant to overfitting (when low noise)	**Must adjust to handle multiclass tasks**
Performs very well in practice	

What we Learned

- Boosting: **Turning a weak learner into a strong one**
- AdaBoost **powerful** and **popular ensemble** method
- **Consistently ranks well** w.r.t. other learning algorithms
- **Each round focus on examples previously misclassified**
- **Different** than **Bagging**
- **Strengths**: simple, few parameters, implicit feature selection, resistant to overfitting (expl. by margin theory)
- **Weaknesses**: outliers/noise, termination, skew/cost-insensitive, must be modified for multiclass problems
- Many interpretations, a **fertile research area**

Thank You!

Questions?